

Analytical Modelling of Thirty Meter Telescope Optics Polarization

Ramya M Anche^a, G C Anupama^a, Krishna Reddy^b, Asoke Sen^c, K Sankarasubramanian^d, A N Ramaprakash^e, Sujan Sengupta^a, Warren Skidmore^f, Jenny Atwood^g, Sivarani Tirupathi^a and Shashi Bhushan Pandey^b

^aIndian Institute of Astrophysics, Koramangala, Bangalore, India

^bAryabhata Research Institute of Observational Sciences, Nainital, India

^cAssam University, Silchar, India

^dISRO Satellite Centre, Bangalore, India

^eInter-University Centre for Astronomy and Astrophysics, Pune, India

^fThirty Meter Telescope Observatory Corporation, Pasadena, USA

^gNational Research Council Canada, Canada

ABSTRACT

The polarization introduced due to Thirty Meter Telescope (TMT) optics is calculated using an analytical model. Mueller matrices are also generated for each optical element using Zemax, based on which the instrumental polarization due to the entire system at the focal plane is estimated and compared with the analytical model. This study is significant in the estimation of the telescope sensitivity and also has great implications for future instruments.

Keywords: Thirty Meter Telescope (TMT), Instrumental polarization, Mueller matrices, Astronomical polarimetry, Analytical modelling

1. INTRODUCTION

Polarimetry is a powerful technique for studying various astrophysical process in celestial objects.¹⁻³ Light is transferred to the polarimeter by the telescope for measuring its polarization. The polarization introduced by telescope because of the multiple reflections from its mirrors is called the instrumental polarization⁴ which increases with the field angle.¹ It is very essential to estimate the instrumental polarization introduced due to telescopes for very high accuracy measurements. Mueller matrix formalism is generally used to estimate the instrumental polarization.^{5,6} It can also be calculated by using M&M codes developed by C U Keller group in Leiden Observatory.^{7,8}

The Thirty Meter Telescope (TMT) (Fig. 1a) is a planned ground-based large segmented mirror reflecting telescope, with optical design⁹ of folded Ritchey- Chretien with diameter of 30m and F/1 focal ratio (Fig. 1b). It is proposed to be built on Mauna Kea in Hawaii, and designed for observations from the near- ultraviolet to the mid-infrared. Polarimetry and Time Resolved Capabilities Working Group (PTRCWG) of TMT has collected a list of 35 potential polarimetric observing programs. This includes studies of wide range of different astrophysical objects, processes, and environments.^{10,11}

In this paper we explain the analytical model formalism and estimation of instrumental polarization (Section 2). We have also estimated Mueller matrices using Zemax (Section 3). The comparison of the results obtained is represented for wavelength range of 0.3-2.5 μm and for telescope pointing at zenith (Section 4). The conclusions are presented in Section 5.

Further author information: (Send correspondence to Authors)

R.M.A.: E-mail: ramyam@iiap.res.in, G.C.A.: E-mail: gca@iiap.res.in

Surface	Radius of Curvature (m)	Conic Constant	Profile
Primary Mirror	-60	-1.00953	Hyperbolic Concave
Secondary Mirror	-6.2	-1.318	Hyperbolic Convex
Tertiary Mirror	NA	NA	Plane Elliptical

Table 1: Specifications of three mirrors of TMT

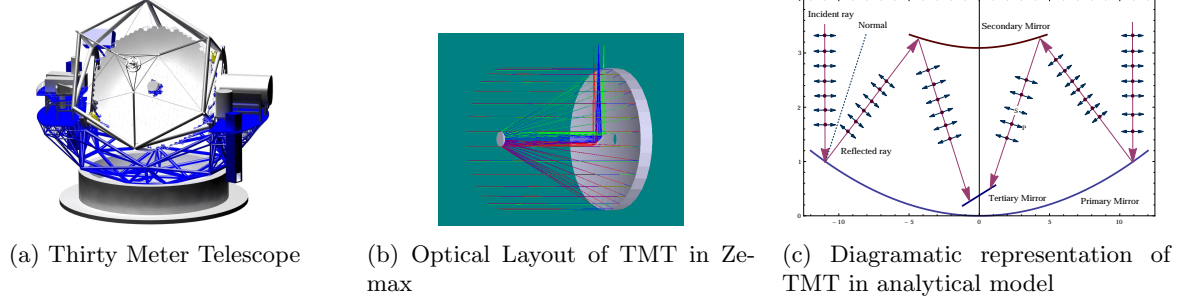


Figure 1

2. ANALYTICAL MODELLING

The analytical model is a ray tracing algorithm from primary to the tertiary (Nasmyth) focus using direction cosines.¹ The specifications of the three mirrors are given in the Table. 1. The list of short forms used for various vectors is given in Appendix A. The primary mirror is considered to be a monolith in our analysis for simplification.

1. Primary Mirror : The equation of a hyperbolic concave mirror with vertex at the origin is given by

$$-\frac{x^2}{a^2} - \frac{y^2}{a^2} + \frac{(z+c)^2}{c^2} = 1, \quad (1)$$

where $a = 1943.6$ and $c = 62959$ are calculated using radius of curvature and conic constant. The rays incident at an angle η with z -axis on the mirror surface will have the direction cosine (DC), $\hat{i} = (\sin \eta, 0, \cos \eta)$, where η is the field angle. The field of view of TMT is 20 arc minute with slight vignetting at the edges of the field. The rays with field angle zero describes a circular symmetry on the surface of the mirror. Any point on this circle can be defined by $(x, y, z) = \left(r \cos \theta, r \sin \theta, c \left(\sqrt{1 + \frac{r^2}{a^2}} - 1 \right) \right)$. The DC normal (\hat{n}) to the surface at this point is given by

$$\hat{n} = \nabla \phi / |\nabla \phi|, \quad (2)$$

where $\phi = 1 + \frac{x^2}{a^2} + \frac{y^2}{a^2} - \frac{(z+c)^2}{c^2}$. The DC of the reflected ray \hat{r} is

$$\hat{r} = 2\hat{n} \cdot \hat{i} \cos(inc) - \hat{i}; \quad (3)$$

where $inc = \hat{n} \cdot \hat{i}$. The diagrammatic representation in (Fig. 1c) shows incident, normal and reflected rays. The electric field vectors of the incident and reflected rays can be resolved into two directions: one perpendicular to the plane of incidence (**S** vector), and the other parallel to the plane of incidence (**P** vector). The direction of **S** vector remains same after the reflection. The **P** vector has pi direction in the incident light, and pr direction in the reflected light. The DC of these vectors are obtained by cross product and normalized as: $\hat{s} = \hat{i} \times \hat{n}$, $\hat{p}_i = \hat{s} \times \hat{i}$, and $\hat{p}_r = \hat{s} \times \hat{r}$.

The TMT mirrors will have a coating similar to Gemini mirrors¹² which is a five layer coating as shown in (Fig. 2). Out of all these layers the two layers 85Å silicon nitride and 1100Å silver layer (Gemini) are

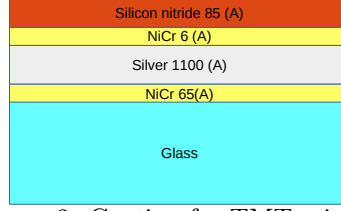


Figure 2: Coating for TMT mirrors

considered in this study. The reflectivities, rp and rs for the two layer coating are calculated as given in Harten, Snik & Keller¹³ explained below.

$$r = \frac{\eta_m E_m - H_m}{\eta_m E_m + H_m} \quad (4)$$

$$E_m = \cos \delta_f + i \frac{\eta_b}{\eta_f} \sin \delta_f \quad (5)$$

$$H_m = \eta_b \cos \delta_f + i \eta_f \sin \delta_f \quad (6)$$

$$\delta_f = \frac{2\pi}{\lambda} \cdot n_f(\lambda) \cdot d_f \cdot \cos \theta_f, \quad (7)$$

where $d_f = 85\text{\AA}$ is the thickness of the Si_3N_4 layer and n_f is the refractive index of Si_3N_4 . n_b and θ_b are the refractive index and the incident angle at bulk metal Silver layer, respectively. n_0 and θ_0 are the refractive index and incident angle for air. θ_f and θ_b can be calculated from the angle of incidence:

$$\sin \theta_0 = n_f(\lambda) \cdot \sin \theta_f \quad (8)$$

$$n_b \cdot \sin \theta_b = n_f(\lambda) \cdot \sin \theta_f. \quad (9)$$

The η_b and η_f in the above equations for p polarization are given by

$$\eta_{b,f,m} = n_{b,f,m} \cdot \cos \theta_{b,f,0}, \quad (10)$$

and for the s polarization:

$$\eta_{b,f,m} = \frac{n_{b,f,m}}{\cos \theta_{b,f,0}}. \quad (11)$$

For an unpolarized incident light, the electric field vector of incident ray does not have any preferential direction. Therefore the electric field vector in p and s directions can be assumed to be $Ep = \frac{1}{\sqrt{2}}$ and $Es = \frac{1}{\sqrt{2}}$ respectively, so that the magnitude of components are equal and the total intensity is equal to 1. The electric field amplitudes of the reflected ray in the p and s directions, are related to their corresponding amplitudes in the incident ray (Ep, Es) by:

$$Rp = rp \cdot Ep, \quad (12)$$

$$Rs = rs \cdot Es. \quad (13)$$

where rp and rs are the reflectivities of the mirror surface. For any electromagnetic wave, it is necessary to define the Stokes parameters to analyze polarization. In the present case, Rp and Rs are the orthogonal components. so the Stokes parameters¹⁴ in $p-s$ coordinate frame can be written as:

$$ip = Rp^2 + Rs^2 \quad (14)$$

$$qp = Rp^2 - Rs^2 \quad (15)$$

$$up = 2RpRs \cos(\delta_p - \delta_s) \quad (16)$$

$$vp = 2RpRs \sin(\delta_p - \delta_s), \quad (17)$$

where δ_p and δ_s are the phase angles of the Rp and Rs components respectively. These Stokes vectors are converted into the cartesian coordinate system, and the values of ixy , qxy , and uxy are determined individually for each ray. These Stokes vectors are integrated over 0 to 2π , and the resultant Stokes vectors for the entire beam $[I, Q, U, V]$ are obtained. The circular component V is not considered in this case. From the I , Q and U values the instrumental polarization percentage is evaluated by:

$$Pol = \frac{\sqrt{Q^2 + U^2}}{I} \times 100. \quad (18)$$

2. Secondary mirror: The shape of the secondary mirror is given by the modified Hyperbola which is shifted up from its vertex, and given by the equation.

$$-\frac{x^2}{a^2} - \frac{y^2}{a^2} + \frac{(z - \tau)^2}{c^2} = 1, \quad (19)$$

$\tau = 6.9196$ is calculated by knowing the vertex c and the distance between the primary and the secondary mirror. The parameters $a = 11.3806$ and $c = 20.1742$ are calculated using the radius of curvature and conic constant. Now, the point of intersection on secondary has to be found out for each ray reflected from the primary. The equation of the ray reflected from the primary can be written as:

$$\frac{x - r \cos \theta}{\hat{r}l} = \frac{y - r \sin \theta}{\hat{r}m} = \frac{z - c \left(\sqrt{1 + \frac{r^2}{a^2}} - 1 \right)}{\hat{r}n} = \lambda \quad (20)$$

(rl, rm, rn) are the (l, m, n) direction cosines of reflected ray from primary mirror, so the above equation can be written as

$$x = \lambda \hat{r}l + r \cos \theta \quad (21)$$

$$y = \lambda \hat{r}m + r \sin \theta \quad (22)$$

$$z = \lambda \hat{r}n + c \left(\sqrt{1 + \frac{r^2}{a^2}} - 1 \right) \quad (23)$$

The (x, y, z) can be calculated by substituting these values in the hyperboloid equation and finding the value of λ . The quadratic equation will result in two roots. Only the positive root is considered as it corresponds to the hyperboloid of our choice. The DC of the normal $\hat{s}n$ and reflected rays $\hat{s}r$ are found from equations (2) & (3) respectively. The angle of incidence and the reflectivities of the mirror surface are found out.¹³ The DC of **S** and **P** vectors after the reflection from the secondary mirror can be obtained by cross product and normalized as: $\hat{s}s = \hat{r} \times \hat{s}n$, $\hat{s}pi = \hat{r} \times \hat{s}s$, and $\hat{s}pr = \hat{s}r \times \hat{s}s$. For each ray reflected from the primary mirror, we have Rp and Rs components. The electric field, amplitudes (sEp, sEs), for the incident ray on the secondary mirror in the **S** and **P** directions are obtained by taking the Rp and Rs components in **S** and **P** directions.

$$Dr1 = \hat{p}r \cdot \hat{s}pi \quad (24)$$

$$Dr2 = \hat{p}s \cdot \hat{s}pi \quad (25)$$

$$Dr3 = \hat{p}r \cdot \hat{s}s \quad (26)$$

$$Dr4 = \hat{p}s \cdot \hat{s}s \quad (27)$$

$$sEp = (Rp.Dr1 + Rs.Dr2) \quad (28)$$

$$sEs = (Rp.Dr3 + Rs.Dr4) \quad (29)$$

The reflected electric field amplitudes from the secondary mirror are found out by multiplying reflectivities of the secondary mirror and the electric field amplitudes incident on the mirror given by equation (13).

Unpolarized rays falling on the primary mirror will get polarized after reflection. Let p be the degree of polarization after reflection from the primary. The rays incident on the secondary mirror consist of two

parts (a) a major unpolarized component, and (b) a small polarized component, maintaining the ratio $(1 - p) : p$ in their intensities. For the unpolarized part, the Stokes vectors (as in 14- 17) are calculated assuming $ups = 0$. For the polarized part, the Stokes vectors are calculated considering $ups \neq 0$. The two sets of Stokes vectors are added and converted to the Cartesian coordinates system. The resultant Stokes vectors are obtained and instrumental polarization percentage at Cassegrain focus (as in 18) is estimated.

3. Tertiary mirror (M3): The tertiary mirror reflects the light coming from the secondary mirror system to the science instruments located on the Nasmyth platforms (Fig. 4). M3 must rotate and tilt as the telescope tilts about the elevation axis and tracks astronomical objects across the sky. The amount of M3 rotation (θ) and tilt (ϕ) is dependent on the location of the active instrument and the zenith angle of the telescope. The coordinate system used in describing M3 motions is given in (Fig. 3) and explained in Table 2 given in the document¹⁵

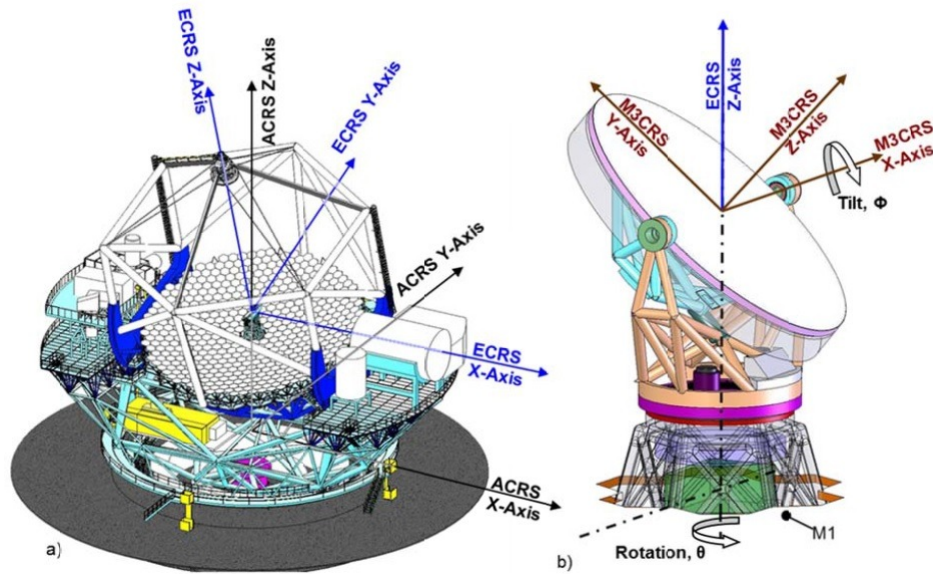


Figure 3: Coordinate systems used in M3 motion

Coordinate System	Origin	X axis	Y axis	Z axis
Azimuth (ACRS)	Intersection of the center of the Telescope azimuth journal and the plane of the telescope azimuth journal	Points East when azimuth angle = 0. Lies in the plane of the azimuth journal. Rotates as telescope rotates in azimuth	Right hand complement to the ACRS X- and Z-axes	Points toward zenith (anti-gravity direction)
Elevation (ECRS)	The intersection of the Z axis of ACRS with the elevation axis of the telescope	Parallel to the X-axis of the ACRS, collinear with the elevation axis of the telescope	Right hand complement to the ECRS X- and Z-axes	Points to zenith when zenith angle=0 and rotates with the zenith angle
Tertiary Mirror (M3CRS)	Identical to ECRS origin	Aligned with ECRS+Y-axis . when M3 rotation angle $\theta = 0$ Collinear with M3 tilt axis	Lies in the plane of the M3 Mirror and is the right hand complement to the M3CRS X- and Z-axes	Normal to M3 surface; points away from the reflective surface

Table 2: Coordinate systems in M3 motion

To calculate the required motions for M3S, the instrument positions must be described. The center of the focal planes of most instruments intercepts the telescope focal plane on the horizontal plane which passes through ECRS and M3CRS origin. This plane shall be called the X-Y Reference plane. The location of the centers of the instrument focal plane positions is described using two angles: the Instrument Bearing Angle (IBA) and the Instrument Elevation Angle (IEA). IBA is the angle from ECRS +X axis to the IBA vector which points from the ECRS origin to the position of the instrument projected on the X-Y reference plane. The IEA is the angle from the IBA vector to the vector which points from the ECRS origin to the

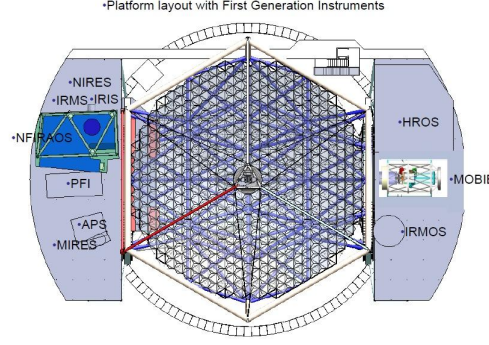


Figure 4: Different Instruments on the Nasmyth platform

center of the instrument focal plane (Fig . 4). The θ and ϕ can be obtained as

$$\theta = \arctan \left(\frac{\cos \zeta \cdot \cos(IEA) \cdot \sin(IBA) - (\sin \zeta \cdot \sin(IBA))}{\cos(IEA) \cdot \sin(IBA)} \right) \quad (30)$$

$$\phi = 0.5 \cdot \arccos(-\sin \zeta \cdot \cos(IEA) \cdot \sin(IBA) + \cos \zeta \cdot \sin(IBA)), \quad (31)$$

where ζ corresponds to the zenith angle of the telescope. It varies from 0 and 65 degrees. θ and ϕ describe the DC of the normal(\hat{tn}) to the M3 mirror given by $(\sin \phi \cos \theta, \sin \phi \sin \theta, \cos \phi)$. The DC of the reflected rays(\hat{tr}) are found from (3). The DC of **S** and **P** vectors after the reflection from the tertiary mirror can be obtained by cross product and normalized as: $\hat{ts} = \hat{sr} \times \hat{tn}$, $\hat{tpi} = \hat{sr} \times \hat{ts}$, and $\hat{tpr} = \hat{tr} \times \hat{ts}$. The Stokes vectors (as in 14- 17) and the percentage of instrumental polarization (as in 18) at Nasmyth focus is modelled.

3. ZEMAX MODEL

The optical layout of TMT is obtained in Zemax as shown in (Fig. 1b). For polarization analysis in Zemax, the “Polarization pupil map” option is used.¹⁰ Zemax will give the polarization parameters like electric field components (real and imaginary); amplitudes, phase and orientation angle of polarization ellipse. Gemini coating is used for the three mirrors. From the electric field amplitudes, Stokes vectors are calculated at each position and they are averaged at the focal plane. The Stokes vectors are calculated for 6 polarized states (horizontal (h), vertical (v), linear 45 (45), linear -45 (-45), right circular (rhc) and left circular (lhc)).

$$I' = Ex^2 + Ey^2 \quad (32)$$

$$Q' = Ex^2 - Ey^2 \quad (33)$$

$$U' = 2.Ex.Ey.cos(\delta_x - \delta_y) \quad (34)$$

$$V' = 2.Ex.Ey.sin(\delta_x - \delta_y) \quad (35)$$

The corresponding Mueller matrix is given by

$$\begin{pmatrix} I'_h + I'_v & I'_h - I'_v & I'_{45} - I'_{-45} & I'_{rhc} - I'_{lhc} \\ Q'_h + Q'_v & Q'_h - Q'_v & Q'_{45} - Q'_{-45} & Q'_{rhc} - Q'_{lhc} \\ U'_h + U'_v & U'_h - U'_v & U'_{45} - U'_{-45} & U'_{rhc} - U'_{lhc} \\ V'_h + V'_v & V'_h - V'_v & V'_{45} - V'_{-45} & V'_{rhc} - V'_{lhc} \end{pmatrix} \quad (36)$$

The instrumental polarization is found by multiplying the Mueller matrix with the stokes vector of unpolarized light [1, 0, 0, 0].

4. RESULTS

The instrumental polarization percentage varies with field angle (over 10'), as shown in the Fig.5. It is found to be 0 for on axis rays at Prime and Cassegrain focus and increases with field angle. At the Nasmyth focus it

Primary				Secondary				Tertiary			
1	0	0	0	1	0	0	0	1	0.012193	9.68e-06	1.94e-06
0	0.99857	0	0	0	0.9993	0	0	0.012198	0.99917	1.86e-05	0.000325
0	0	0.99866	0	0	0	0.99934	0	9.21e-06	0.000115	0.88725	-0.45871
0	0	0	0.99724	0	0	0	-0.99864	2.65e-06	-0.0003	0.45905	0.8865

Table 3: Mueller matrix at 600nm obtained by Zemax for on axis rays at Prime, Cassegrain and Nasmyth focus.

is found to be 1.23% for on axis rays when the θ and ϕ for the tertiary mirror is 0 and 45 degrees respectively. The results obtained by analytical method and by Zemax (Mueller matrices) are compared in the Fig.6 over the wavelength range $0.3 - 2.5\mu\text{m}$. The polarization caused by the telescope is less in the optical and infrared range compared to near ultraviolet region.

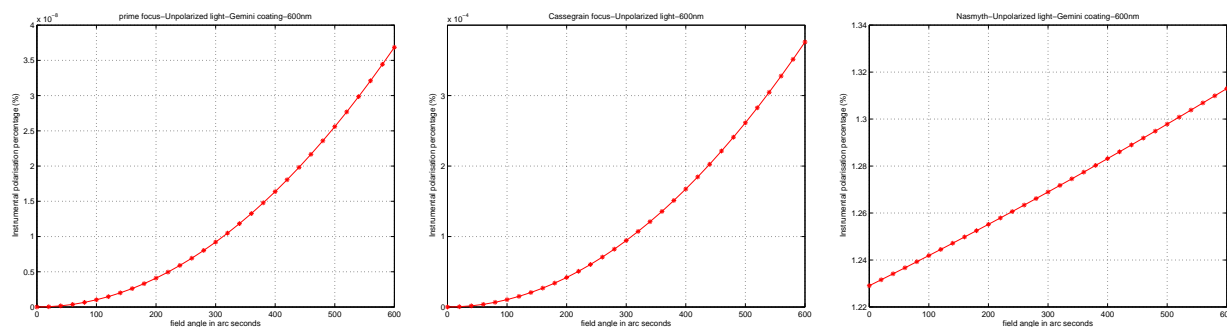


Figure 5: Instrumental polarization percentage at Prime, Cassegrain and Nasmyth focus by analytical model

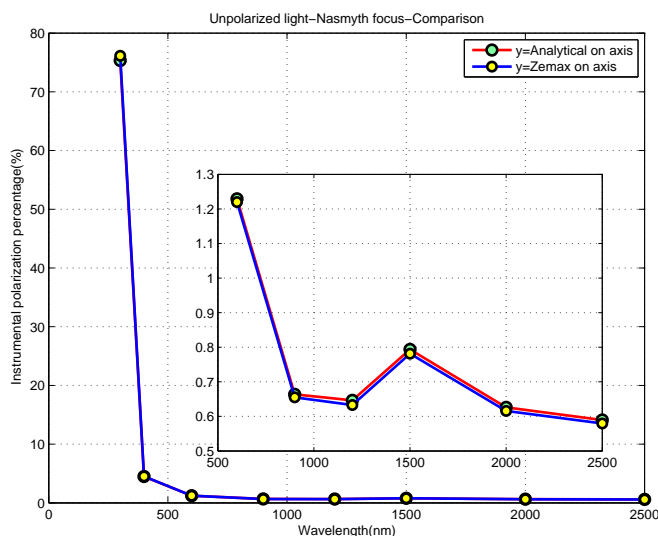


Figure 6: Comparison of Analytical and Zemax for 0 field over wavelengths

5. CONCLUSIONS

The instrumental polarization percentage is found at the Nasmyth focus by both analytical model and Zemax and the corresponding values are in good agreement. From the results we see that the tertiary mirror is the main source of polarization. The polarization science programs require the wavelength range from $0.3 - 2.5\mu\text{m}$. But from our studies, we conclude that it is difficult to carry out polarimetry at $0.3\mu\text{m}$ since the reflectivity is less and the percentage instrumental polarization is very high.

APPENDIX A. THE LIST OF SHORT FORMS USED FOR VARIOUS VECTORS:

\hat{i} =Direction cosine of the incident ray on the primary mirror.
 \hat{n} =Direction cosine of the normal to the primary mirror.
 \hat{r} =Direction cosine of the reflected ray from the primary mirror.
 $\hat{p}s$ =Direction cosine of the **S** with reference to primary mirror.
 $\hat{p}i$ =Direction cosine of the **P** in the incident ray on primary mirror.
 $\hat{p}r$ =Direction cosine of the **P** in the reflected ray on primary mirror.
 $\hat{s}i$ =Direction cosine of the incident ray on the secondary mirror.
 $\hat{s}n$ =Direction cosine of the normal to the secondary mirror.
 $\hat{s}r$ =Direction cosine of the reflected ray from the secondary mirror.
 $\hat{s}s$ =Direction cosine of the **S** with reference to secondary mirror.
 $\hat{s}p i$ =Direction cosine of the **P** in the incident ray on secondary mirror.
 $\hat{s}p r$ =Direction cosine of the **P** in the reflected ray on secondary mirror.
 $\hat{t}i$ =Direction cosine of the incident ray on the tertiary mirror.
 $\hat{t}n$ =Direction cosine of the normal to the tertiary mirror.
 $\hat{t}r$ =Direction cosine of the reflected ray from the tertiary mirror.
 $\hat{t}s$ =Direction cosine of the **S** with reference to tertiary mirror.
 $\hat{t}p i$ =Direction cosine of the **P** in the incident ray on tertiary mirror.
 $\hat{t}p r$ =Direction cosine of the **P** in the reflected ray on tertiary mirror.

REFERENCES

- [1] Sen, A. K. and Kakati, M., “Instrumental polarization caused by telescope optics during wide field imaging,” *Astron. Astrophys. Suppl.* **126**, 113–119 (Nov. 1997).
- [2] Tinbergen, J., [*Astronomical Polarimetry*] (Aug. 2005).
- [3] Clarke, D., [*Stellar Polarimetry*], John Wiley & Sons (2009).
- [4] Keller, C. U., “Instrumentation for astrophysical spectropolarimetry,” in [*Astrophysical Spectropolarimetry*], Trujillo-Bueno, J., Moreno-Insertis, F., and Sánchez, F., eds., 303–354 (2002).
- [5] West, S., “Polarimetry with multiple mirror telescopes,” in [*1986 Astronomy Conferences*], 50–59, International Society for Optics and Photonics (1986).
- [6] Tinbergen, J., “Accurate Optical Polarimetry on the Nasmyth Platform,” *Pub. Astron. Soc. Pac.* **119**, 1371–1384 (Dec. 2007).
- [7] de Juan Ovelar, M., Diamantopoulou, S., Roelfsema, R., van Werkhoven, T., Snik, F., Pragt, J., and Keller, C., “Modeling the instrumental polarization of the VLT and E-ELT telescopes with the m&m’s code,” in [*SPIE Astronomical Telescopes+ Instrumentation*], 844912–844912, International Society for Optics and Photonics (2012).
- [8] Ovelar, M., Snik, F., Keller, C., and Venema, L., “Instrumental polarisation at the nasmyth focus of the E-ELT,” *arXiv preprint arXiv:1312.6148* (2013).
- [9] Williams, E., “TMT telescope optics,” *TMT.OPT.PRE.09.099.DRF03* (2009).
- [10] Atwood, J., Skidmore, W., Anupama, G., Anche, R. M., Reddy, K., and Sen, A., “Polarimetric analysis of the thirty meter telescope (TMT) for modeling instrumental polarization characteristics,” in [*SPIE Astronomical Telescopes+ Instrumentation*], 915013–915013, International Society for Optics and Photonics (2014).
- [11] Skidmore, W., Atwood, J., Manjunath, R., Reddy, K., Anupama, G., and Sen, A. K., “Polarimetric analysis of the thirty meter telescope (TMT) for modeling instrumental polarization characteristics,” in [*American Astronomical Society Meeting Abstracts*], **224** (2014).
- [12] Anne, D. and Maxime, B., “Polarization study of gemini’s coatings,” *Gemini technical report* (2008).
- [13] Van Harten, G., Snik, F., and Keller, C., “Polarization properties of real aluminum mirrors, i. influence of the aluminum oxide layer,” *Polarization* **121**(878), 377–383 (2009).
- [14] Collett, E., “Polarized light. fundamentals and applications,” *Optical Engineering, New York: Dekker,—c1992* **1** (1992).
- [15] TMT corp, *Design requirements document for tertiary mirror system (M3S)*, TMT.opt.drd.07.006.rel29 ed. (jul 2012).